Applications of the Triple-grid Technique to the Inversion of DC Resistivity Data

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Abstract

We present our triple-grid technique for the inversion of direct current (dc) resistivity data from a users point of view. It makes use of unstructured tetrahedral or triangular meshes and the finite element method. Three meshes are used: a parameter mesh describes the modeling domain. The forward calculation is done on the secondary field mesh, whereas the primary potentials are obtained by a nonrecurring simulation on a refined primary mesh. By the latter we are able to separate the effect of the topography and the subsurface. We exemplify this by a 2-d profile measurement across a waste dump.

Unstructured grids prove to be particularly efficient for geometry or large geometry contrasts. The Plesse castle shows heavy topography in form of an altitude jump. On the Cuxhaven site a lot of dc soundings have been made with electrode separations varying from 0.5 m to 5000 m. We show that the technique can also be applied to closed geometries in form of a tree section. All these problems cannot be treated by structured grids since the geometry would cause the inverse problem too big.

An important issue is the incorporation of a-priori information into the inversion. The inversion of underwater data shows how the existing geometry can be involved in the technique. A second example treats the combination of different methods. The knowledge of a layer boundary from refraction seismics is used to obtain a much clearer image of the subsurface.

Introduction

The rapid development of multi-electrode devices allows for efficient investigation of the subsurface in three dimensions. 3-d inversion algorithms have been presented by Park and Van (1991) or Ellis and Oldenburg (1994) using the Gauss-Newton method or conjugated gradients, respectively. The forward calculation is done by finite differences (Zhang et al., 1995) or finite elements (Sasaki, 1994). The latter is able to involve topography (Yi et al., 2001; Pain et al., 2003). However, the use of structured grids restricts the complexity of the surface. Moreover, it produces unnecessarily much free parameters.

We present an approach using unstructured tetrahedral or triangular meshes. Firstly it allows for a resolution-dependent parameterization which helps to keep the problem small. The forward calculation is done by a secondary potential method (Coggon, 1971) with finite elements. For details on the forward and inverse procedure we refer to (Rücker et al., 2006) and Günther et al. (2006), respectivly.

The generalized inversion scheme is displayed in Fig. 1. We used the topographical information for generating the three meshes. First, the primary potentials are calculated and used to derive the apparent resistivities. In the inversion loop (rectangle) the model is changed until convergence is reached.

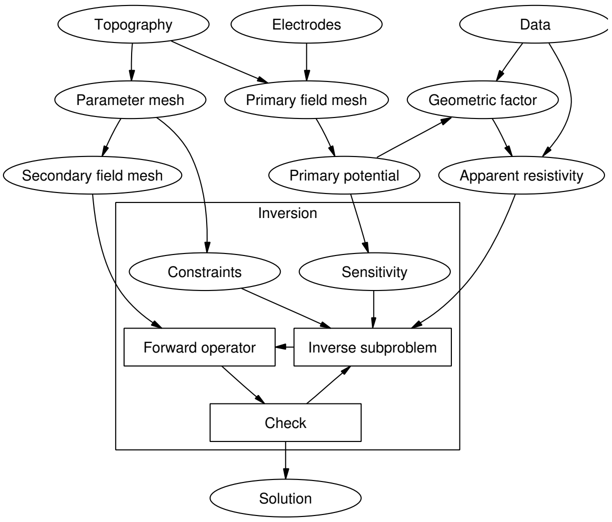
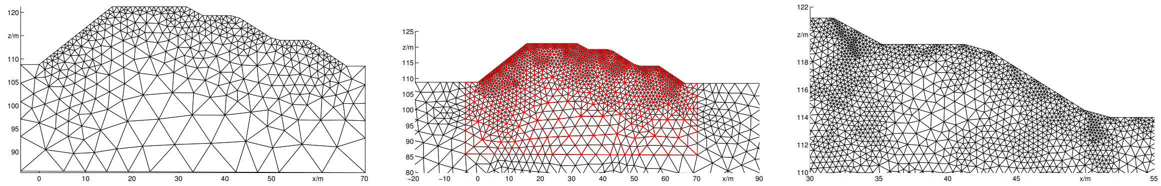


Figure 1: Inversion scheme of the triple-grid inversion

Demonstration of the technique

The following data are descended from the investigation of an abandoned mining dump. We regard one of the profiles across the dump with 5 m electrode distance. A Wenner pseudo-section has been measured to investigate the function of the hard pan as sealing layer and the interior of the dump.

Figure 2 shows the three meshes used in inversion. They all describe the topography of the 14 m high and 65 m broad dump. The parameter mesh (a) is included in the secondary field mesh (b), whereas the primary field mesh (c) is an independently obtained refined mesh.

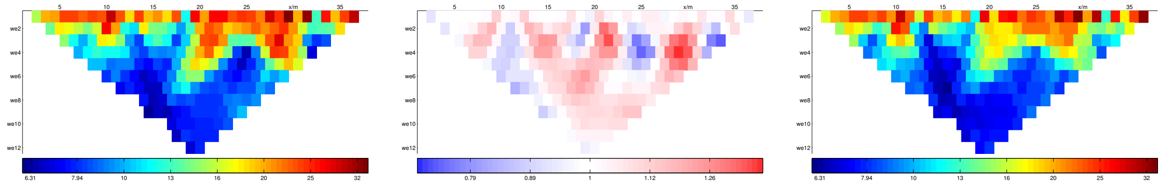


(a) (b) (c)

Figure 2: The three meshes of inversion, (a) the parameter mesh, (b) the secondary field mesh and (c) a section of the primary field mesh

In Fig. 3 we study the effect of the topography. First we use the analytical (flat-earth) geometric factor for determining the apparent resistivities (a). We see anomalies but can not distinguish if they are due to topography or subsurface. In (b) the pure topography effect is displayed showing similar structures as in (a), particularly at the topographical edges. Finally, (c) displays the apparent resistivities obtained by simulated geometric factors. The anomalies are much weaker and not affected by the topography anymore.

The final inversion result is imaged in Fig. 4. We observe a resistive hard pan with varying thickness. In the interior the dump body shows up with low resistivities.



(a) (b) (c)

Figure 3: Topography effect: apparent resistivities with analytical geometric factors (a), simulated geometric factors (c) and pure topography effect (b)

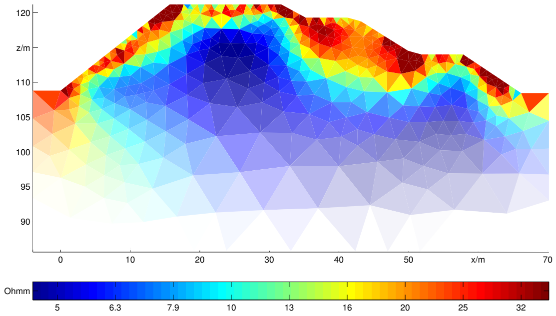


Figure 4: Inversion result showing the resistive hard pan and the conductive interior of the dump

Geometric flexibility

Steep topography - Plesse Castle

The Plesse Castle is a popular castle near Göttingen. Archaeologists search for an ancient well in the inner ward which is investigated by dc resistivity measurements. Figure 5 shows a photograph and the profile layout. The particular difficulty is a jump of up to 4 m in the topography on the ascent that cannot be handled by block-oriented grids.

All 9 profiles have an electrode distance of 1 m. As a compromise between resolution and signal strength we chose a combination of Wenner-α and -β data. The topography inside and outside the measuring area was recorded accurately. In Fig. 6 the final inver-

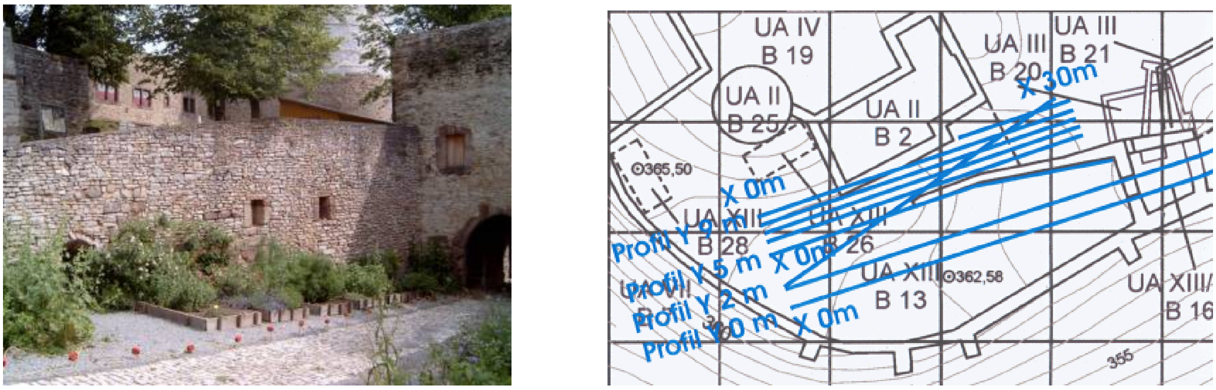


Figure 5: Photo (left) and profile layout (right) of the survey

sion result is shown in form of two iso-bodies of < 50 and $> 250\Omega$m. The walls at the boundaries and the existing vault behind the central wall show up by high resistivities. Also, beneath the tree on the upper plateau high resistivities are to be seen. Behind we see a large anomaly of low resistivity that we interpret as the well filled with loose material and a higher water content.

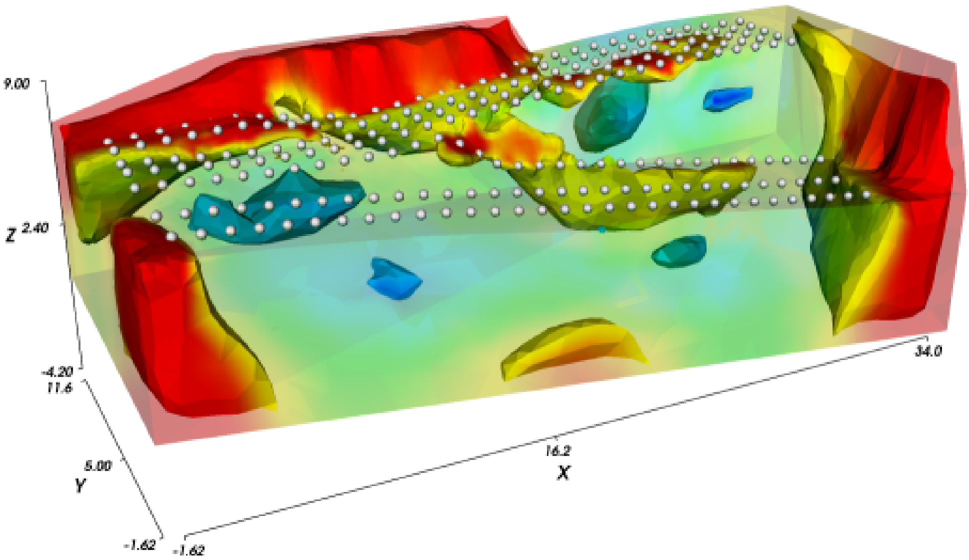


Figure 6: 3-d model of the inversion result

Geometry contrasts - Cuxhaven site

In the North Sea region glacial channels are present that have been built up by floated melting water. Since later being filled up with sediments the so-called Buried Valleys are important for groundwater questions. In the recent decades many geophysical investigations were made in order to investigate the structure of the valleys. So a lot of dc resistivity soundings with the (full or half) Schlumberger array have been measured in the region of a known buried valley near Cuxhaven. However, a one-dimensional interpretation is difficult because the underground is three-dimensional.

The problem of a 3-d inversion routine are the large geometric contrasts, generated by logarithmically growing electrode distances from 0.5 m up to 5000 m. Thus the huge area of 10×12 km is covered very irregularly. A block-oriented discretization would create lots of parameter cells that can not be handled by a standard pc. With the presented approach the number of parameters could be restricted to 60 000.

The final model can be seen in Fig. 7 showing several sections and an iso-surface of 13 Ωm. The cell sizes vary from a few meters at the sounding positions to kilometers near the boundary. At the surface, higher resistive sediments are present. In northeastern direction we see the conductive salt-water intruding from the north sea. In the southwestern part the course buried valley can be tracked.

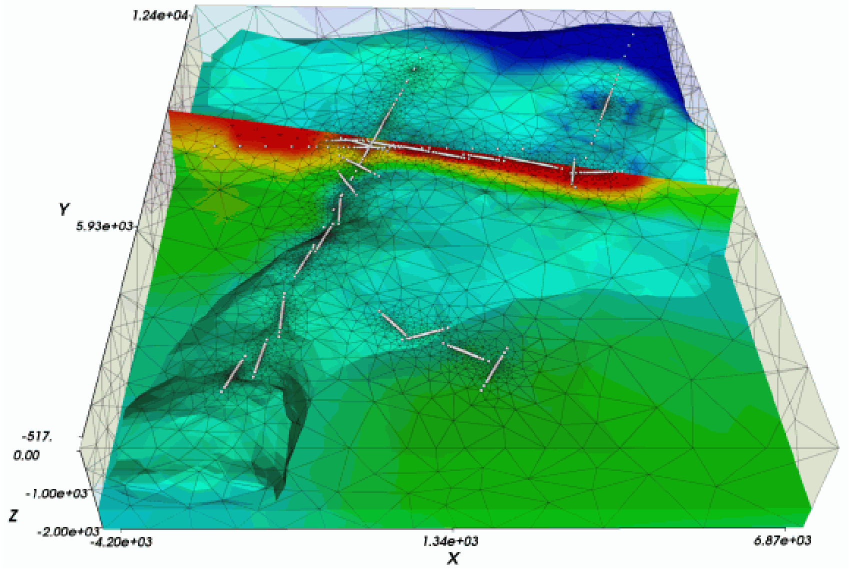


Figure 7: 3-d model of the inversion result

Tree tomography

Direct current resistivity measurements can also be applied to objects of closed geometry. A typical application is the investigation of trees by nondestructive impedance measurements. The resistivity can provide valuable information on the interior, e.g. about decay or damages.

Often the tree shape deviates from a simple circle and has thus to be incorporated. Figure 8 shows an example of a hollow lime tree that has been cut after the measurements. Thus, we are able to validate the inversion results.

The inversion may be extended to 3-d inversion by measuring on different levels. For example, it may be investigated how precipitation affects the resistivity in ground probes or a model tank.

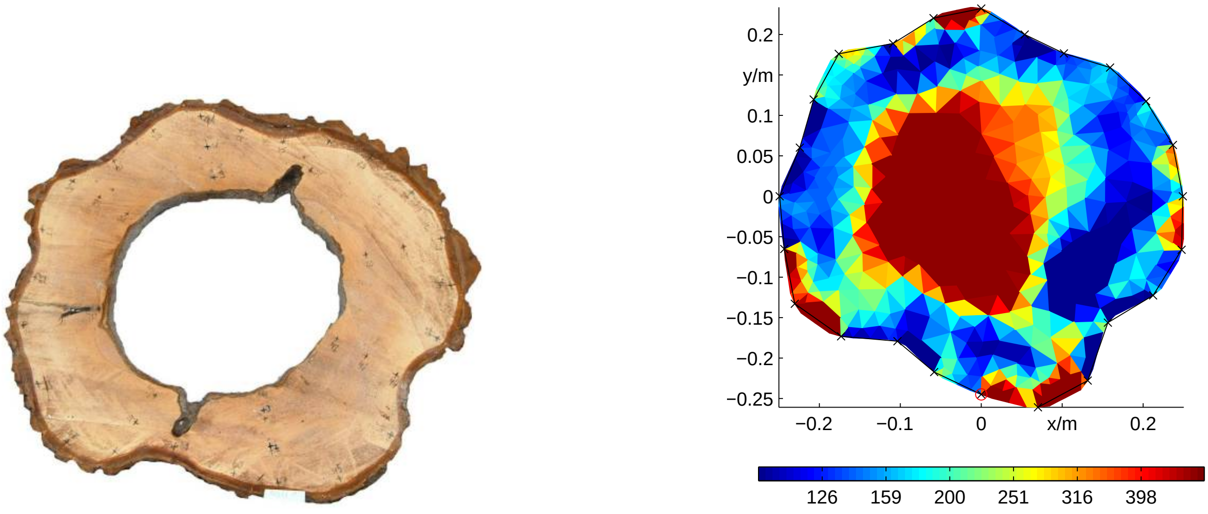


Figure 8: Section of the hollow lime tree (a) and inversion result (b) clearly showing the resistive anomaly

Sophisticated constraints

Underwater survey - Lake Feldungel

Underwater measurements are a challenge for both data acquisition and inversion. The course of the ground and the electrode positions have to be determined and introduced into the inversion algorithm. Also the water resistivity must be considered in the forward calculation. In our approach we include the water layer in the secondary mesh, but not in the parameter mesh.

The following example was measured in the Feldungel lake near Osnabrück. In the scope of the background of the Teutonian War archaeologists are interested in the ancient natural scenery. The lake has been formed by a depression and preserves the sedimentation history of the past 10 000 years.

We installed an electrode line with 2 m electrode distance on ground of the lake. A Schlumberger section and a Wenner-β section have been measured and used for the inversion. The water resistivity was assumed to being 22.5 Ωm throughout the lake. Furthermore, we used this value as logarithmic barrier to restrict the resistivities to being above. Since we expect more or less layered structures, we used anisotropic constraints, i.e., the more the normal vector of the boundary edge tends vertical, the less weight is given on the individual constraint. The final result is displayed in Fig. 9.

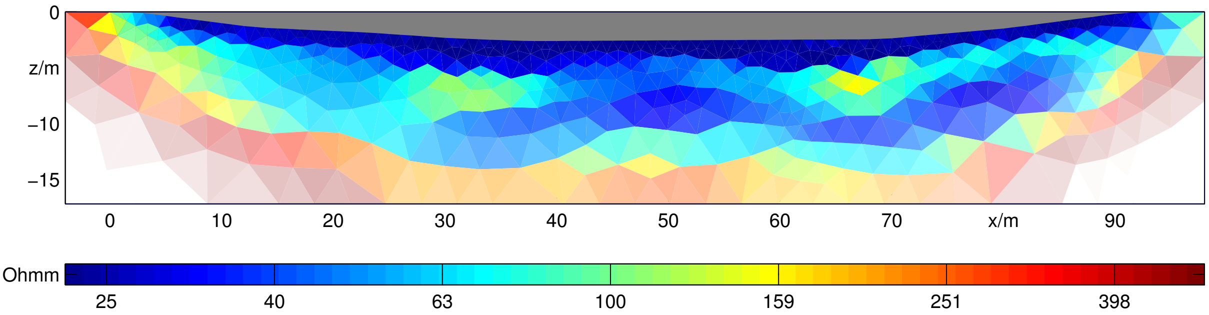


Figure 9: Inversion result of the Feldungel survey, the water (grey) is part of the forward meshes, but not of the parameter mesh

Directly below the ground we see resistivities similar to the water, since the mud has a high water content. In a depth of up to 14 m below the water table the surface of the depression can be observed. At medium depths we can see sedimentation structures that confirms the assumption of the lake being constituted of two part separated by a barrier before the water table rose.

Seismics-constrained inversion

In many investigations, geophysicists use different physical methods in order to control the ambiguity of the interpretation. Additional information, e.g. from bore holes or other data, can serve as soft or hard constraints. The following example was measured by the K-UTec GmbH, Sondershausen. Aim of the survey was to detect the top edge of the bedrock with seismic refraction and dc resistivity measurements. The refraction data exhibit good quality and have been interpreted by a two-layer case with varying layer boundary.

The resistivity data were inverted and displayed in Fig. 10 together with the refractor (left). Obviously the velocity boundary can be seen in terms of resistivity, too. Therefore we introduced the refractor as a known boundary in the model generation and allowed for sharp resistivity contrasts at these positions. The result is shown on the right side of Fig. 10.

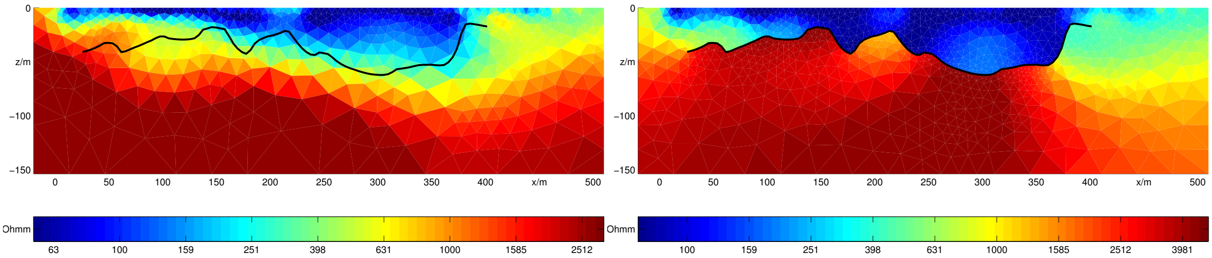


Figure 10: Inversion result without (left) and with (right) the refractor as known boundary

Now we obtain a much clearer image of the subsurface, particularly in the central part, where we have almost a two-layer case. On the right hand side the resistivity sees the bedrock at depth and does not accept the given boundary which is certainly to steep to be resolved. At the left hand side there might be an additional unit that is invisible in terms of velocity.

The proceeding may be extended to bore holes, where punctual layer boundaries help to obtain sharper images. An further improvement can be done by a combined tomography in form of a joint inversion.

Conclusions

We presented different applications for the triple-grid technique for dc inversion. They prove the great flexibility of integrating a complicating geometry by the use of unstructured tetrahedral or triangular meshes. The use of constraints will be formulated more general in order to allow for an easier use of a-priori information. A further improvement may be achieved by the use of inversion methods that avoid the storage of the Jacobian matrix, i.e. conjugated gradients (Ellis and Oldenburg, 1994) or quasi-Newton methods (Haber, 2005). The forward calculation is intented to apply an adaptive mesh refinement to increase accuracy. Finally, it should be analyzed, whether the methods can be carried forward to other geophysical methods or the joint inversion of different data.

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